Forum: Comment l'art peut-il venir en aide à l'enseignement des mathématiques?

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1 Background

This informal essay is based on an introductory statement I gave in the forum 'How art can contribute to the teaching of mathematics?' It is not intended to be a closely reasoned statement of my 'philosophical position' on the teaching of mathematics (I have no such position).

I would like to start by describing some of my background and experience related to the topic of the forum and explain how a mathematician came to be using ideas based on art (and computers) in his teaching.

1.1 *Prism*

About eleven years ago, I started to develop a computer program called *prism* (PRogram for the Interactive Study of Maps). While the program arose out of research into symmetric dynamics [1,5], the main use of *prism* was for the design, creation and colouring of symmetric planar patterns. Examples of patterns designed using *prism* are shown in the book *Symmetry in Chaos* [4] written jointly with Marty Golubitsky in 1992.

Over the years, *prism* has continued to develop and now includes a multiplicity of algorithms for all seventeen of the wallpaper (planar crystallographic) patterns as well as the forty-six 2-colour wallpaper patterns [2]. Characteristically, the designs created using *prism* exhibit a richly complex and (of course!) symmetric structure. Colouring – closely related to the underlying dynamics – is not routine and requires considerable input from the designer. Some examples of recent designs created using *prism* may be found in in my article in the Maubeuge Proceedings [3][1].

1.2 Symmetry in design: An disciplinary course for art students

About four years ago, Angela Patton (Department of Art, UH) suggested that I develop an interdisciplinary course on patterns, designs and symmetry based on *prism*. Although I had largely developed *prism* for personal use, I was intrigued as to how art and design students would respond to an 'art' program from a mathematician. There was also the challenge of communicating

[1] See also my web site `http://nothung.math.uh.edu/~mike/`.

some of the underlying mathematical ideas about geometry and symmetry to 'math-unfriendly' students. In the event, I developed a new course 'Patterns, Designs and Symmetry' for Junior and Senior level students at UH. The course has been given three times and is now part of the regular course schedule at UH. The year 2000 course has a mixture of art, graphics, photography, architecture, and mathematics students.

Implementation and results One of the aims in giving the 'Patterns, Designs and Symmetry' class was to expose students to the mathematical concept of symmetry and thereby enhance their visual perception and design skills. This had to be done in a context where it was not acceptable to develop the mathematics in a formal didactic way. Indeed, many of the students were not only alarmed by the prospect of being taught by a mathematician but some were also nervous about the prospect of learning about and using computers. Equations and formulae were largely banned from the class and characters like 'X' and 'Z' were introduced for their symmetry properties rather than as symbols to be used in algebraic manipulation. Geometry was, however, strongly emphasized. By the end of the course, students were expected to be able to find the glide reflection symmetries of a planar pattern, distinguish, for example, between wallpaper patterns of type **p3m1** and **p31m**, and identify the symmetries of a polyhedron (including the regular stellated polyhedra). The approach that I followed was a mix of theory and practice. Practice was vital. Students designed symmetric patterns, created informative posters and made solids. For example, some of the students made symmetrically coloured models of the regular stellated dodecahedra. As a text we used the book by Washburn and Crowe [7] supplemented by my own notes and websites such as the geometry junkyard (URL: www.ics.uci.edu/˜eppstein/junkyard/). About 40% of the course was spent in a computer lab. At the outset, I attempted to sensitize the students to symmetry. In Houston, this was easily accomplished by initiating an active and ongoing study of the symmetry of (car) wheels. Using wheels as a model, we discussed the different types of (bounded) symmetry in the plane. Symmetry was introduced as an operational property: an object is symmetric if you can pick it up, move it around and put it back in the same space as it was originally – but with a different orientation. Along the way, it was possible to introduce some ideas from algebra – to help with the composition of symmetries – and to point out how order of composition matters. Concurrent with the investigation of bounded symmetry, students were learning how to log on to a computer and use software packages. Their first project using *prism* was to design, and later colour, a bounded symmetric image in the plane. The successful completion of this design was an essential, highly motivational, part of the learning process – a movement from theory (of symmetry) to producing a design. And the designs were often very attractive. (See the URL: http://nothung.math.uh.edu/˜patterns/indexart.html for some examples from the 1999 class.)

1.3 Mathematics through Art?

In 1999, I organized a seminar 'Symmetry, patterns and designs' for the inaugural year of the *Houston Teacher's Institute* (HTI). The HTI is part of a national demonstration project led by the Yale-New Haven Teachers Institute and supported by the DeWitt Wallace-Reader's Digest Foundation. (For further information, see the URL: www.cis.yale.edu/ynhti.) Roughly speaking, the idea of the HTI is that a group of about twelve teachers (the 'Fellows') participate in a collegial seminar led by a faculty member at UH. Each Fellow producing a curriculum unit related to the topic of the seminar (see the HTI website, URL: www.uh.edu/hti, for more details).

Most of the Fellows in my seminar were mathematics teachers in local middle or high schools. I used *prism* as a motivational tool tool to introduce ideas about symmetry and chaos. Part of the seminar involved each Fellow producing their own graphic. Some of these graphics were used as the basis for a quilt pattern which was used as the inside cover for the published set [6] of curriculum units for the entire 1999 Houston Teacher's Institute.

Implementation When planning the seminar, I had to keep in mind that content had to be interesting and stimulating for the teachers. Practically speaking, that meant that what we discussed had to have the potential for *practical application* in a high or middle school classroom. In particular, it was essential that the material could be used to facilitate the understanding of geometry (or algebra – if that were the teacher's specialty). The situation for teaching general mathematics courses at college level is broadly similar. It is not simply a question of making mathematics appealing or user-friendly. The message has to have content – and that content must lead to an enhanced understanding of the world that the student perceives.

In the event, none of the teachers had any background in symmetry. This was an advantage as it meant there were no preconceptions to overcome. Just as for the art class, the topic of the symmetry of wheels was interesting to the teachers – but now because they knew that this was one of the topics teenagers could get excited about (we did not discuss the possible relation between symmetry, beauty and sexual attractiveness – though that came up in some of the curriculum units). Another area of great potential was the use of symmetry in textiles – again dress is important to the target group. Architecture and art provided other possibilities. One of the most interesting suggestions made was to combine art and mathematics classes as a way of allowing the students to use ideas about symmetry in their art – this for a group of students who were almost totally alienated from mathematics and who, on good days, spent their time in mathematics classes drawing (or doodling). Overall, I felt the class was quite successful and the teachers produced units that far exceeded my initial expectations (the units may be seen on the HTI website).

2 Conclusions and comments

2.1 Alienation from mathematics

Since I started teaching in the United States, I have often encountered a strong alienation from mathematics among university students. One explanation for this alienation is the custom in the United States of requiring students to complete 'core' courses in 'college algebra' if they have not already done these classes in high school. Many of these so-called 'core' courses are unmotivated terminating courses that are intensely disliked by the students (correctly disliked in my opinion). In my view the forced learning of college algebra at university is an optimal strategy for the development of hostility towards mathematics in the general population.

I believe another reason for the negative attitude towards mathematics lies in the attitude of elementary school teachers towards mathematics. Indeed, if elementary school teachers are hostile towards mathematics, that attitude will surely propagate to their students. I have certainly found strong negative attitudes amongst (prospective) elementary school teachers. For example, a few years ago I took a summer class on geometry at UW-Madison for a group of prospective elementary school teachers. When I met the class for the first time, I found they were uniformly hostile towards mathematics and resentful that they had to take the class. They also told me – correctly, I believe – that they felt the class (syllabus) was irrelevant to their classroom needs. In the event, I developed a syllabus that focussed on developing classroom materials that the teachers could use. I felt that the students completed the class with a strong positive attitude to at least part of mathematics.

2.2 The right solution is that there is no 'right' solution

In my opinion, there is no 'right' solution for the teaching of mathematics at school or college level. For example, it is well known that while some students respond well to visual materials, other do not and instead feel more comfortable with an abstract/algebraic approach.

I believe the enthusiasm and involvement of the teacher is the *key* factor in the success of a new approach to the teaching of mathematics. Pilot/experimental programs are frequently successful just because they have the active involvement and support of the teachers who developed the program. I have been fortunate at UH in that I have been allowed and encouraged to develop experimental programs. If instead, I had *directed* others to develop and teach programs in 'art and mathematics', these programs would probably have failed because of the lack of commitment and involvement on the part of the teachers carrying out my instructions. In short, I am sceptical of a 'top-down' approach to the teaching of mathematics that is based on edicts from on high (whether administration or senior academics) as to what the teachers ought to be teaching. Rather, I believe that initiatives should come from the teachers and should be supported (facilitated) by the administration. Even if projects eventually fail, the effort and enthusiasm for mathematics shown by the teachers will propagate to the students.

A particular obsession amongst mathematicians is that of 'error'. By this I do not mean numerical error, but rather the fear that mathematical concepts will not be accurately presented to the students. When this view dominates, it is difficult to allow any teacher initiatives – such as the preparation of curriculum units I described above. In extreme form, the result is that every detail of the syllabus has to be approved by government appointed committees of 'experts' – lest the children or students be exposed to an erroneous idea or concept. The reality is that people – especially children – are incredibly robust to error and are quite capable of handling conflicting or incomplete conceptual information. Better that the children learn to enjoy and appreciate geometry, algebra and statistics than that they be alienated from mathematics for ever by the joys of rote long division, factorization or the nuances of the law of the contrapositive.

In brief, we need to trust our teachers and faculty and support and facilitate them in their efforts to develop new ways to teach mathematics.

References

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