The Thom Seminary: Its Genesis

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Born in 1923, René Thom cheerfully went through all the stages of a brilliant academic career. His mathematical work is in line with that of Russian and American mathematicians. Thom published his first note in the Comptes-Rendus at the age of 26. The content of this note, « Sur une partition en cellules associée à une fonction sur une variété » (« On a partition into cells associated with a function on a manifold ») foreshadows that of all his later mathematical work. The properties of the function he uses announce his interest in the theory of singularities of differentiable maps. The partition into cells of a manifold refers to its interest in topology, in particular the classification of differentiable manifolds.

He published his thesis two years after this first note in the Comptes-Rendus, and obtained the Fields Medal in 1958, at 35 years of age, for his work in algebraic topology on the classification of differentiable varieties using significant groups. The use of the assembly of varieties by cobordism, that is to say their association according to compatible boundaries, allowed him to widen the field of the results already known.

In one of our conversations, around 1975 I think, he revealed to me spontaneously, blushing, that the mathematician Gaston Julia had advised him to study singularities. I made the mistake of not asking him about when this suggestion came about. He worked in this direction as an extension of the work of Marston Morse and Hassler Whitney. He once told me that he considered Whitney to be the most important mathematician of his century¹. This was of course a personal point of view.

I no longer remember how I was informed of a presentation by Thom, at the mathematics department of Orsay, on a Thursday in early June 1968.

It was afternoon, the sky was all blue and the the sun was shining. I was sitting on the left side, facing the desk in front of which Thom was presenting some of his views on biology and linguistics. Thom spoke of this using the elements of his theory of course. The text appeared in number 4, December 1968, of the journal *l'Âge de la Science (Age of Science)*, under the title *Topologie et Signification (Topology and Meaning)*, and it specified the content of the part of the presentation devoted to linguistics.

I was fascinated by the cleverness and wealth of this presentation, whose mathematical roots were somewhat magical for me at the time, delighted if not joyful because I encountered in it the geometric attitude of my own approach to linguistics. At the time, I had already written my first article on linguistics entitled *Sur une for-*

¹ One of the points of connection between Thom and me was the fact that Whitney was also the founder of the theory of matroids, the subject of my main thesis.

malisation de la grammaire française simplifiée² (On a formalization of simplified French grammar). I relied on the fact that an important part of speech, through the simulation activity performed by the brain, reflects the spatial organization of objects. I found in Thom's underlying presentation the same point of view.

My article appeared in 1970 only in *Etudes de Linguistique Appliquée*. This delay in publication allowed me to complete the original writing simply by adding Thom's name in the introduction, without giving any bibliographical reference, and in particular concerning him since the approach, of an algebraic nature, was very different from his, which was centered on a form of analytical representation of certain verbs. The following is the introduction to this text³:

SUR UNE FORMALISATION DE LA GRAMMAIRE FRANÇAISE SIMPLIFIÉE

0. Introduction.

Dans l'essai qui suit, on présente une formalisation abrégée de la grammaire française simplifiée, en tenant compte de quelques-unes de ses propriétés sémantiques.

Il nous est apparu que certaines règles fondamentales de la construction de nombreux langages, y compris le langage mathématique, rentraient dans le cadre de ce formalisme. Il présente donc un intérêt pédagogique et méthodologique.

Son choix repose sur le fait qu'un langage a pour but de décrire les objets dans l'espace et dans le temps, ainsi que leurs interactions et mouvements. Ce fait, pressenti par N. Chomsky, semble assez important pour avoir, indépendamment de nous, retenu l'attention de R. Thom, et lui permettre d'introduire de nouveaux modèles mathématiques en linguistique.

So sharing with Thom the same spatio-temporal understanding of language, I came to see him at the end of his presentation, told him of my feeling of complicity with his geometric vision of language. He invited me to come and meet him at 11 a.m. at IHES, the last Saturday of the month.

That day, we resumed an appointment for the Saturday of the new school year, again at 11 a.m., at the IHES. We were thus alone for at least two years, after which other people joined, the first of them being Pierre Delattre†. Many who attended what became a regular seminar are present here, but there were also many other participants, more or less episodic: Delattre's two collaborators at the CEA, Hyver and Del-

² Accessible on http://arpam.free.fr/FGFS.pdf

³ On a formalization of simplified French grammar. In the following essay, we present an abbreviated formalization of simplified French grammar, taking into account some of its semantic properties. We learned that certain fundamental rules for the construction of many languages, including mathematical language, fell within the scope of this formalism. It is therefore of pedagogical and methodological interest. Its choice is based on the fact that a language aims to describe objects in space and time, as well as their interactions and movements. This fact, sensed by N. Chomsky, seems important enough to have, independently of us, caught the attention of R.Thom, and allowed him to introduce new mathematical and linguistic models.

forge, my late friends, the linguist Maurice Coyaud⁴ (†), who knew almost all the languages and dialects of Asia!, the mathematician Jean-Pierre Duport-Rosand⁵ (†), author of a very beautiful tale *La princesse bleue* (*The Blue Princess*), inspired by the notion of chreod, and author of a great application of CT to hemophilia, the philosopher of science Jean Largeault⁶ (†) of course, the morphologist, biologist and physicist Yves Bouligand ⁷(†), the philosopher Kristof Pomian, the international lawyer Alain Laraby, the neurobiologist Pierre Etevenon, whose guinea pig I was for a while, and several others whose names you know, who are now absent.

Thom remained practically silent during our one-on-ones sessions where a relaxed atmosphere of perfect understanding reigned between us. We didn't talk about biology in any way. I explained my ideas to him, most of which, written at the CNRS center in Aussois in September, was the subject of my mimeographed text of October-November 1969 entitled On the cave wall (Sur le Mur de la Caverne⁸). The content of this essay provided the material, and the first part of Volume 1, and Volume 3 of Topologie et Perception⁹ (Topology and Perception). In the essay, the announcement of the expected publication of Stabilité Structurelle and Morphogenèse (Structural Stability and Morphogenesis) appears in the bibliography. The book was to appear in 1970. It was finally published in November 1972, and André Avez had guessed perfectly why the last chapters bear the discreet traces of my comments. They are not very visible, even to me now, because Thom had integrated them into his vision of things at the time. I would not be surprised if the Appendix on the notion of object was added following our meetings. Thom had only given me the proofs of these last chapters. We can see, in the preface he wrote for *Topologie et Perception*, the expression of our intellectual sympathy and no doubt a certain form of acknowledgment of debt.

What I learned from Thom therefore only comes, as for everyone, from reading some of his texts and from his public presentations. In particular, I wrote up his lessons given on Monday afternoons at the IHES throughout the 1970-1971 academic year, and entitled *Modèles mathématiques de la Morphogenèse*¹⁰ (*Mathematical Modèles of Morphogenesis*). Thom told me he would thank me, but... These texts do not

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⁵ Maître de Conférences à l'Université de Savoie

⁶ Professeur à l'Université Paris 12

⁷ Président après Thom de la Société Française de Biologie Théorique

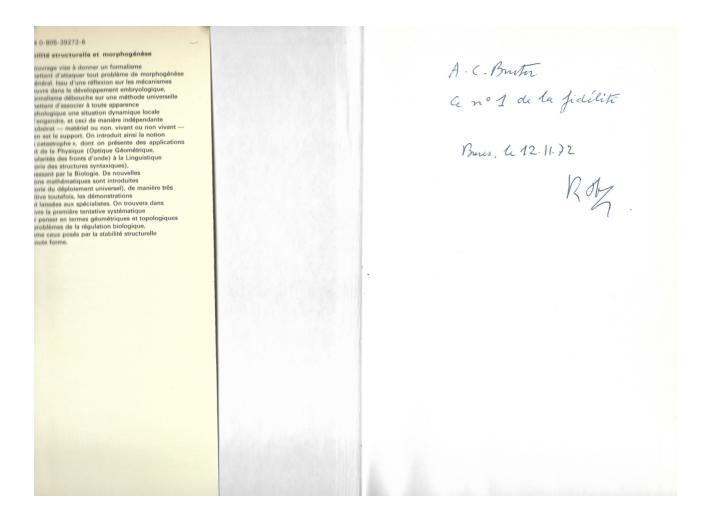
⁸ http://arpam.free.fr/SMC.pdf

⁹ http://arpam.free.fr/DP.pdf

¹⁰ http://arpam.free.fr/MMM.pdf

appear in the works of Thom published by mathematicians. They served as the basis for the publication of the *Lezione Fermiane* included in his book edited by Flammarion. The introduction where the problem of explanation appears, for example, is mine, there is also the use of the term extremality, still unknown to dictionaries. I did not bring anything on the mathematical level, I learned it. I did, however, contribute to the formulation of his model. The first two chapters were not modified. Only the second part of the third chapter had been reviewed by Thom, as at the end of page 32: « I gave ... ».

What I have retained, in an essential way, from this long contact with the writings of Thom, is the importance of the notion of singularity (introduced by Cauchy), and that of stability. Let me quote a few lines from the "Advice to the Reader" that appears in my book titled *Energie et Stabilité*¹¹ and Stability:



"A word about the focus in this text on stability. It comes from my meetings with Thom. First of all, I am grateful to him for having directed me to André Avez, who died recently, quietly regarding the mathematical community. I would like to pay homage to him here. By assigning me the subject of my second thesis, Avez introduced me to the theory of dynamical systems; the notion of stability occupies an eminent place. At that time, influenced by the popularity of structuralism, the notion of structural stability occupied a pro-

¹¹ Accessible on http://arpam.free.fr/ESC.pdf

minent place in people's minds. Thom, in his writings, speaks essentially only of this one. I felt the need to go beyond this restrictive point of view. I add that this was all the more necessary since structural stability is by no means generic, as Smale has shown.

I have made this notion of stability a sort of operational law, analogous to certain fundamental laws of physics, such as those of gravitation or those of Maxwell, a law of a metaphysical nature given the observed universality of its field of application. I call it "Plato's principle" (cf *The Symposium*, 207d) and state it today in this form: *Every object strives to persevere in its "self", across space and time*. If Pascal had read Plato's Banquet, he would have found the answer to one of his questions. Taking up and developing the content of the first part of *Topology and Perception* from the angle of the Mother Ideas¹², a book nearing completion reveals the extent of the impregnation of the principle and its driving force in all of the activities of the natural world.

I would like to thank Sharon Breit-Giraud who generously corrected my first translation of the original French text.

¹² The Principe of Stability, an Essay on the Incarnation of the Notion of Stability within the Pantheon of Mother Ideas <u>Cambridgescholars.com</u>, 2021 <u>https://www.cambridgescholars.com/product/978-1-5275-6857-0</u>