

## FOREWORD

This volume presents the proceedings of a colloquium inspired by the former President of the French Mathematical Society, Michel Herve. The aim was to promote the development of mathematics through applications.

Since the ancient supports the new, it seemed appropriate to center the theoretical conferences on new subjects.

Since the world is movement and creation, the theoretical conferences were planned on mechanics (movement) and bifurcation theory (creation). Five aspects of mechanics were to be presented, but, unfortunately, it has not been possible to include the statistical mechanics aspect. So that only four aspects are presented:

- Classical mechanics (Hamiltonian, Lagrangian, Poisson) (W.M. Tulczyjew, J.E. White, C.M. Marle).
- Quantum mechanics (in particular the passage from the classical to the quantum approach and the problem of finding the explicit solution of Schrödinger's equation) (M. Cahen and S. Gutt, J. Leray).
- Fluid mechanics (meaning problems involving partial differential equations. One of the speakers we hoped would attend the conference was in Japan at the time, however his lecture is presented in these proceedings.) (J.F. Pommaret, H.W. Shi)
- Mathematical "information" theory (S. Guisasu)

Traditional physical arguments are characterized by their great homogeneity, and mathematically expressed by the compactness property. In such cases, there is a kind of duality between locality and globality, which allows the use of the infinitesimal in global considerations.

In the papers, infinitesimal methods appear through the use of infinitesimal operators (in particular differential forms and Lie groups), and through the use of Taylor's series expansion (jet bundles: the use of this language is the most convenient in the search for solutions of partial differential equations on Riemannian spaces). Global considerations appear through the use of global energy functions, and extremal or variational principles (see the

paper by L. Nirenberg). None of these principles is really well understood.

Three aspects of bifurcation theory are embraced:

- Bifurcation in ordinary differential equations (the lecturer centered his talk on his work on some special Hopf bifurcations. The results are included in the reviewed paper written by a colleague who could not attend the colloquium (W. Broer).
- Numerical methods in bifurcation theory tied to the Lyapunov-Schmidt procedure (J. Rappez).
- Bifurcation in partial differential equations involving the the Lyapunov-Schmidt procedure and singular theory (M. Golubitsky).

At this point, a feeling arises that group considerations play an increasing role in the study of bifurcation phenomena. These can be understood as the results of bifurcations of group (and pseudo-group) actions.

Monge's method, expanded by W. Shi, is used by W. H. Shi to treat a non-trivial example. The physical significance of this example is criticized by J. Leray; however the method remains strong despite this criticism. Besides I would like to point out that we are not always sure which mathematical formulation of a physical problem is the best.

This volume presents a large number of open problems concerning topological methods which are useful to show the influence of the topology of the space of solutions induced by the functional equations to be solved and the nature of the boundary conditions. The arising or vanishing of topological obstructions are obviously bound to shock and bifurcation phenomena. In any case, the Monge-Shi method has to be handled with care in order to take into account Borel's phenomenon (a well-chosen variational coefficient induces the non-analyticity of the unique solution of the partial differential equation with analytical data) or the turbulence phenomena. The problem already posed by small denominators in classical mechanics suggest that the role of number theory will increase in the study of refinements in bifurcation theory.

The last paper presented on bifurcation theory (M. Golubitsky) mainly concerns the B nard problem. This paper is followed by an illustrative article (S. Fauve and A. Lichaber) on experiments showing turbulence phenomena and chaos. At that moment, the homogeneity of the physical state is somehow perturbed but through renormalization, a discrete modelization applies.

Numerical analysis can only use discrete models. Non-standard analysis can be seen as a convergence technique from the discrete to the non-discrete. The next paper (C. Lober and C. Reder) uses this recognition to solve some classical partial differential equations. Automata defined on finite sets (time excepted) can but admit periodic or quasi-periodic regimes. Extensions of such models through non-standard analysis might preserve some periodi-

cities and be convenient models for some natural phenomena, including chemical systems.

The next paper (P. Dousson) is devoted to the mathematical study through quasi-autonomous "ordinary" differential or chemical systems satisfying Wei's axiomatic equations.

The volume concludes with papers (F.A. Grünbaum, M. Kleman, Y. Bouligand) on new mathematical applications to subjects which have recently been developed: tomography on the one hand, liquid crystals on the other hand. If tomography leads to the development of analysis, liquid crystals have given rise to the first useful applications of algebraic topology - through homotopy groups - to the study of physical structures. Liquid crystals appeal also to new mathematical studies in Euclidean, Riemannian, and particularly hyperbolic geometry.

The talk given on a use of catastrophe theory leading to a positive inhibition of hemophilia was not written because of health problems suffered by the author. This is regrettable since, for the first time, the treatment of a disease until now incurable, has been made possible through the use of mathematics. However, an audio-cassette, prepared by J.P. Duport, is available from him.

To end with, I would like to suggest the study of three physical problems, all related to morphogenesis.

(1) Study experimentally and mathematically the physical morphogenesis introduced by Leduc at the beginning of the century.

Note that in the study of biological morphology involving membranes, differential geometry based on surface metrics (Cartan's metrics) should have an advantage over differential geometry based on line metrics (Riemann's metrics).

(2) Study experimentally and mathematically the trajectories of air molecules in a real balloon inflated by blowing it up. (Of course, this problem can be refined by introducing different kinds of local constraints on the shape of the balloons, and by inflating the balloon in different ways.)

(3) Study experimentally and mathematically the evolution in a convex body of sound waves created by a tiny shock on the boundary or inside the body. (The problem interests not only acousticians, but also morphogenesisists; think of the problem of fecundation and very early embryology.)

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C. P. BRUTER