Control and topology

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Two morphological aspects of control are examined: the first one leads to the interdependence between control, stability and symmetry; the second is suggested by the study of the position in space of the control system inside the controlled plant. Two structural remarks are made; the extent of their generality is proved on extremely different examples. The problem is studied within the framework established by the geometrical transduction of the above mentioned properties. One shows why the dynamic systems are to be considered as framework. Some mathematical conjectures, suggested by observations, are given in detail.

INTRODUCTION

The notion of control is difficult to master. Modern attempts of accounting the presence of control mechanisms, based on the cybernetical approach, succeeded initially in building models of an essentially analytical nature. These models, more or less in a geometrical form nowadays, are treated within the framework of the control theory [1]. This theory considers only the functional aspects of the control, leaving aside altogether the morphological aspects.

Neverthelles, the connections between form and function are deep. To see them in a simple way it is sufficient to examine the possible use of the different objects which can be made form a ball and a tube. One should not neglect the interest raised by the morphological properties of various control elements. Anyway, science has to explain them, to justify their presence, their role, their composition, their mode of operation.

The observations upon which we are going to focus our attention might appear trivial and naive. Too simple phenomena are usually held as too obvious to examine and taken for granted without discussion which is not our point of view.

Firstly, we believe that progress in science takes place through the phenomena situated at the boundary of knowledge. Secondly, we believe that it is extremely important to understand completely the so called simple phenomena, since we think that there exists a lattice-like construction of objects. The lowest level among them on this scale consists of most "elementary" objects and — at the same time — the most stable, which allows them to serve as constituents of more complex objects, objects to which they lend their essential characteristics. These characteristics have to be discovered and understood.

1. FIRST OBSERVATION

Our first remark will be: (R): — full control is achieved, not only dynamically, but also morphologically, transversally to the controlled object.

This means the following: if F is a geometrical representation of a control system, B is a geometrical representation of the controlled object, F, defined in R^3, meets B, defined in R^3 in such a manner that the sum of the dimensions of the spaces tangent to F and B in a point of their intersection is zero or equal to 3. Obviously, F and B are geometrical stylisations, often robust, which cannot, in general, be mistaken for the realities they pretend to represent. Moreover, similar to the manner in which the reflection one obtains from the objects creates the image we have in front of our eyes, one cannot be sure that the chosen approach is fit to describe all the possible levels of analysis of the sensitive world.

Since, in the usual environmental space, we see only the edges of objects, the standard observation in R^3 is that of a portion of a sphere S^2 which crosses transversally an arc of curve I. I plays the role of the control system F; S^1 or S^2 play the role of the controlled object B.

One can give several examples which illustrate these geometrical representations: i) the envelope of the family group represented by the outer walls and the roof of the house in which that family lives may be regarded as a portion of a sphere. All the communication paths which enter this house (chimneys, water pipes, gas pipes, electrical wires, telephone lines, the roads and lanes) can be represented, in a first approximation, as arcs of curve. But the communication channels are also control channels.

ii) A country, a town are usually represented by a circle drawn on a plane. Again, all the roads which ensure the control of these communities cross transversally the boundaries of the territories on which these societies are establised.

iii) One can consider as a sphere S^2 the envelope of the living bodies. The cilia, the flagella, the hair, the thorns, the hoofs, the pseudopodes, the limbs, contribute to ensure the equilibrium between the living beings and the external medium. In a first approximation one can represent these organs as arcs of curve.

These naive considerations [2] have been suggested to me by an attempt to represent the duality in matroids theory. According to this theory, every matroid accepts two representations, one being the dual of the other; from here — the idea that one of the representations refers to the control system of a natural object, while the other representation corresponds to the complementary part, within the object, of the control system. If the needs of simplifying the proof require it one can make the difference between the control system and the controlled object, although one knows that these two entities cannot be separated. Thus, the use of a mathematical Janus is obvious. One may remark that most of optimisation problems imposed on such Januses are solved by working, alternately, on the primary and on the dual representations: therefore, one notices an "oscillation" which propagates itself along a closed loop system.

It is worth observing that these remarks on the transversal control could have suggested elements of possible explanation regarding the crossing of nervous fibres, the role of particular neurons of the brain, the activity of particular neurons of the retina in conjunction with the appearance of some geometrical illusions [3].

As long as the remark (R) is founded, it provides a guide in explaining the lay-out and the functional role of some parts of the body, be they internal or external. One can see the interest raised by the morphological considerations, even if they are gross approximations, directed by a safe mathematical framework, which implies, and partly justifies, the analogies.

2 SECOND OBSERVATION

The control system has its contribution in maintaining in a steady state the object of which it is a constitutive part. An object which easily becomes unstable is regarded as a badly controlled one. Therefore, one can say, at least from a theoretical point of view, that the control theory is a part of qualitative dynamics in the most general meaning of the word. One can then ask the question: which is the simplest way of inducing stability?

The observation of natural morphologies provides again, the answer. In order to render stable an unstable system with respect to a certain action group G, one should nest it in a larger object, the morphology of which is invariant with respect to a subgroup g of G. If n is the dimension of the Lie group G, g — in the simplest cases — is the group of symmetries of order n. The elementary cases are those for which g is isomorphic to $z/2z$ or $z/3z$.

The case $g = z/2z$ corresponds to the very many situations in which Nature proceeds simply by duplication to stabilise the object. One might think that in such cases the energy of the object is defined by a Lagrange operator $L(x)$, G-invariant, and that Nature tries to realise an universal stable unfolding of $L(x)$, say $L(x, u)$, such that $L(x, u)$ is g-invariant, g being the smallest possible group which is compatible with the stabilisation of $L(x)$.[7]

It is important to notice that, by using this viewpoint, the symmetry offered by natural objects is no longer one of their *a priori* properties, but the result of necessary stabilisation in the space-time environment of the previous constituents.

The first obvious consequence of this stabilisation is, from the morphological point of view, the presence of the double structures ($g = z/2z$). Since an object shows too poor structural stability qualities in order to last for a long time in its environment, Nature duplicates at least, this object. The typical example is that of the duplication of circuits and security services in human organizations. In this way, each great actor or singer at a theatre or opera has a replacement; policemen walk around two by two, the chiefs of states have a vice-president or a successor, able to ensure the continuity of power in time and space. Basically, the principle of reproducing a species, a sort of quasi-duplication displayed in time, is that of ensuring the stability in space and time of this species. And the symmetry of our body is again the consequence of a stability which is necessary in a three-dimensional environment.

We do not have data about the genesis of the phenomena of duplication, and we might regret it, so the dynamics of the processes which control them escape our understanding. It would not appear as wise to think that a unique kind of branching was used to build the double structures. The Gedanken-experimenten cannot represent more than vague indications. Assume that a coin stands on its edge on the table; one is faced with an unstable system. There are two methods of stabilising it. According the first, one should make the coin thick enough in order to form a cylinder. The second method consists of using a second coin of some size and a rod, connecting the two coins with the rod in a fashion similar to a dumb-bell. The second system can be obtained from the first by decreasing continously the cylinder diameter, keeping constant the diameters of the extremities, until the second system is obtained. This process shows how one can build in a continuous manner and starting from a dynamically unstable system $D = (V, X)$ a system dynamically stable, consisting of two copies D' and D'' of D and of the control system τ transversal to D' and D''.

3. THIRD OBSERVATION

After emphasizing the elementary morphological properties of the natural objects we would like to say something about the dynamics which support them, insisting on the properties of stable rhythmicity which characterize the well-controlled objects.

The electro-magnetic vibrations, the cycles of the cosmic space have a high degree of stability. All the living cells show signs of self-rhythmicity. Two centuries after La Mettrie [5] and his contemporaries, the importance of multiple biological rhythms which characterize any living structure, seems to be taken in consideration again.

The experiments, though simple, performed at Bordeaux by M. Pacault [6] on chemical systems show the very large complexity of the elements used for producing oscillations. The results of such experiments show that it is an illusion for the moment to try to work with quantitative models which take into account all the physically noticeable parameters yet. One should use global models.

In the case of simple oscillations (a stable singular point being subject to Hopf's bifurcation in order to produce a stable limit cycle), which is the case of a great number of experiments made by chemists and thermodynamicians which is also the case of nervous and heart oscillations, it looks advisable to follow Zeeman model [8], based on Thom's potential, in x^4. In the case of multiple oscillations, Zeeman model should be extended by using as a basic variation that which one can define by deriving Thom's potential of co-rank 1 but with a higher co-dimension. Thus, geometrical schemes of the distribution of vigilence states among the various observed types, as well as dynamical models of spatio-temporal oscillations can be given.

By the way, it should be interesting to examine the birth mechanisms, the effects of the stabilisation of such rhythms at the level of social structures. A certain number of corresponding cycles show very little stability or can be represented by attractors in spiral which, eventually, converge towards the stable limit cycles.

With respect to the present state of observation, the class of dynamical systems with tor and solenoids as attractors would appear important. This class might be dense in the C^1 topology. The fact that horseshoes are excluded does not at all prejudged their reality, however, these horseshoes have not been empirically observed.

It must also be noted that present investigations of all natural systems indicate that these system are developping in such a way that the attractors of their representative dynamics become tori of larger and larger dimensions.

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